Mind the Gap: Boundary Conditions for Turbulence Modelling

Nils T. Basse

Independent Scientist

Gothenburg, Sweden

Outline

- My CV
- Introduction
- Eddy viscosity models
- Turbulence intensity:
  - Research findings
  - Recommendations
- Conclusions and outlook

My CV: Education

- 1993-2002:
  - Physics studies, Niels Bohr Institute, University of Copenhagen
  - 1999-2002:
    - Ph.D in optical measurements of fusion plasma turbulence [1.5 yr DK/2 yr DE]
  - 1996-1998:
    - M.Sc in physics [1 yr DK/1 yr UK]
  - 1993-1996:
    - B.Sc. In physics [DK]

My CV: Jobs

- 2019-present:
  - Volvo Cars [SE], thermofluid modelling and simulation of electric drivelines
- 2016-2019:
  - Danfoss [DK], simulations and acoustic measurements of valve flow
- 2011-2016:
  - Siemens [DK], modelling and simulation of flowmeters
- 2006-2011:
  - ABB [CH], modelling and optical measurements of circuit breaker flow
- 2002-2005:
  - MIT [USA], optical measurements of fusion plasma turbulence

Introduction

- Overall topic:
  - Turbulent flows:
    - Modelling of measurements
    - Computational fluid dynamics (CFD) simulations [Roache 1972]
- Focus on boundary conditions (BCs):
  - Measurement example:
    - Well-conditioned pipe flow
  - CFD example:
    - Reynolds-averaged Navier-Stokes (RANS):
      - Two-equation eddy viscosity models (EVMs):
        - Standard $k - \varepsilon$ model

[Pope 2000]

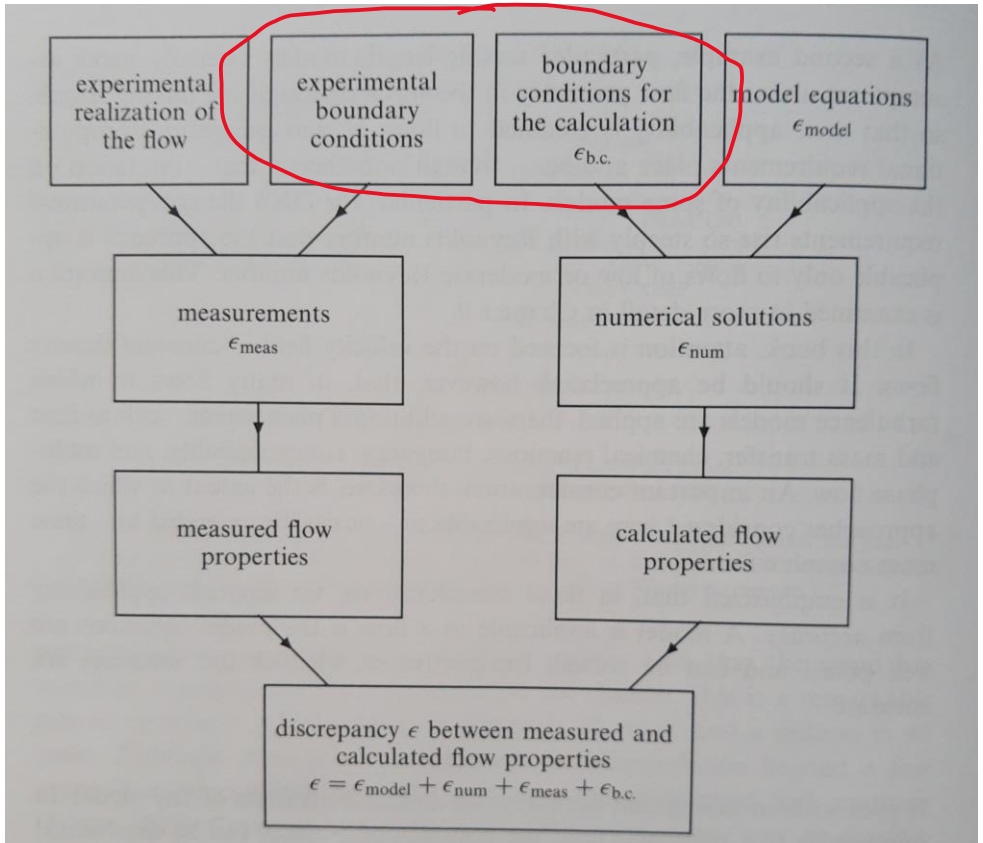


Fig. 8.2. A chart illustrating that the discrepancy ϵ between measured and calculated flow properties stems from inaccuracies of the model, ϵ_{model}; numerical errors, ϵ_{num}; measurement errors ϵ_{meas}; and discrepancies in the boundary conditions, $\epsilon_{b.c.}$. (The equation given for ϵ is merely suggestive: the errors do not add linearly.)

Standard $k - \varepsilon$ model

- Model constants in use today [LS 1974]
- RANS models no longer an active research topic
- But widely used by CFD practitioners

→Gap

2.2. Recommended Constants and Functions

At high Reynolds numbers, the transport equation for ϵ may be expressed:

$$\frac{D\epsilon}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial x_k}\left[\frac{\mu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_k}\right] + \frac{C_1 \mu_t}{\rho}\frac{\epsilon}{k}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\frac{\partial U_i}{\partial x_k} - C_2\frac{\epsilon^2}{k} \qquad (2.2\text{–}1)$$

a form which was first developed and used in the Imperial College group by Hanjalić [17]. Equation (2.2–1) together with a similar one for the turbulence energy, k:

$$\frac{Dk}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial x_k}\left[\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_k}\right] + \frac{\mu_t}{\rho}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right)\frac{\partial U_i}{\partial x_k} - \epsilon \qquad (2.2\text{–}2)$$

enables the turbulent viscosity μ_t to be found from equation (2.1–13) or its equivalent in terms

* The values of C_1 and C_2 depend on the choices for m and n.

B.E. Launder, D.B. Spalding, The numerical computation of turbulent flows 275

of ϵ (rather than l); thus:

$$\mu_t = C_\mu \rho k^2/\epsilon\ . \qquad (2.2\text{–}3)$$

According to the recommendations of Launder et al. [19], made after extensive examination of free turbulent flows, the constants appearing in equations (2.2–1)–(2.2–3) take the values given in table 2.1:

Table 2.1
The values of the constants in the $k \sim \epsilon$ model

C_μ	C_1	C_2	σ_k	σ_ϵ
0.09	1.44	1.92	1.0	1.3

The above constants have been found appropriate to plane jets and mixing layers. Slightly different values from those quoted have hitherto been adopted in the calculation of flows near walls; but there is reason to suppose that, for these flows also, the values in table 2.1 would lead to as satisfactory preditions as obtained with those originally employed.

Eddy-Viscosity Transport Modelling: A Historical Review

- Chapter by K.Hanjalić and B.E.Launder in [Runchal 2020]

5 The Current Usage of Two-Equation EVMs

While today (in 2020) a large proportion of commercial CFD computations are made with eddy-viscosity models such as those outlined above, several model originators (including the present authors), sensitive to the limitations of that closure level for flows subject to complex strains or force fields, have thrown the weight of their efforts to higher order schemes. In such approaches, closed transport equations (or sometimes truncated algebraic versions thereof) are devised for the second moments (the Reynolds stresses and turbulent heat or mass fluxes) and, sometimes, for higher order moments too. Such schemes lie beyond the scope of the present article, but the books by Wilcox [81], Pope [60], and Hanjalić and Launder [27] provide an account of the development of these types of model and, particularly the last of these, present an array of applications.

But two-equation linear eddy-viscosity models remain the workhorse of the CFD industry and while, to many with a deep knowledge of turbulence and its modelling, their continuing popularity remains a mystery, the reasons are not hard to see. Such models rarely give major convergence problems and, even for complex flow configurations, they can be applied at modest computational cost on an acceptable timescale. Moreover, the accuracy of the predictions they deliver is often good enough to guide industrial design or to trace the cause of a malfunction in some operating plant or machinery. Indeed, it may be said that weaknesses that would seem to be dramatically damaging in a strictly two-dimensional shear flow may turn out to be far less influential in three-dimensional flows—which includes the majority of cases for which commercial CFD is employed.

[HL 2021]

VIII. CURRENT TRENDS: HIGHER-ORDER MODELS VERSUS EVMS

The authors have for decades been advocates of using second-moment closure models in the firm belief that, despite their greater demands on computer resources, they captured the physics more faithfully and, thus, should reproduce more accurately complex turbulent flows over a greater range of configurations and conditions than eddy-viscosity schemes. However, the expectation that, with advances in computer hardware, our view would gradually prevail has not in fact come about. Indeed, the opposite has happened: the ever widening of the CFD community and the expansion in the scope of CFD applications has brought about greater demands on CFD-code vendors to ensure their codes' robustness (i.e., certainty to converge to a result), computational economy, and "user friendliness"—demands which inevitably favor simpler models.

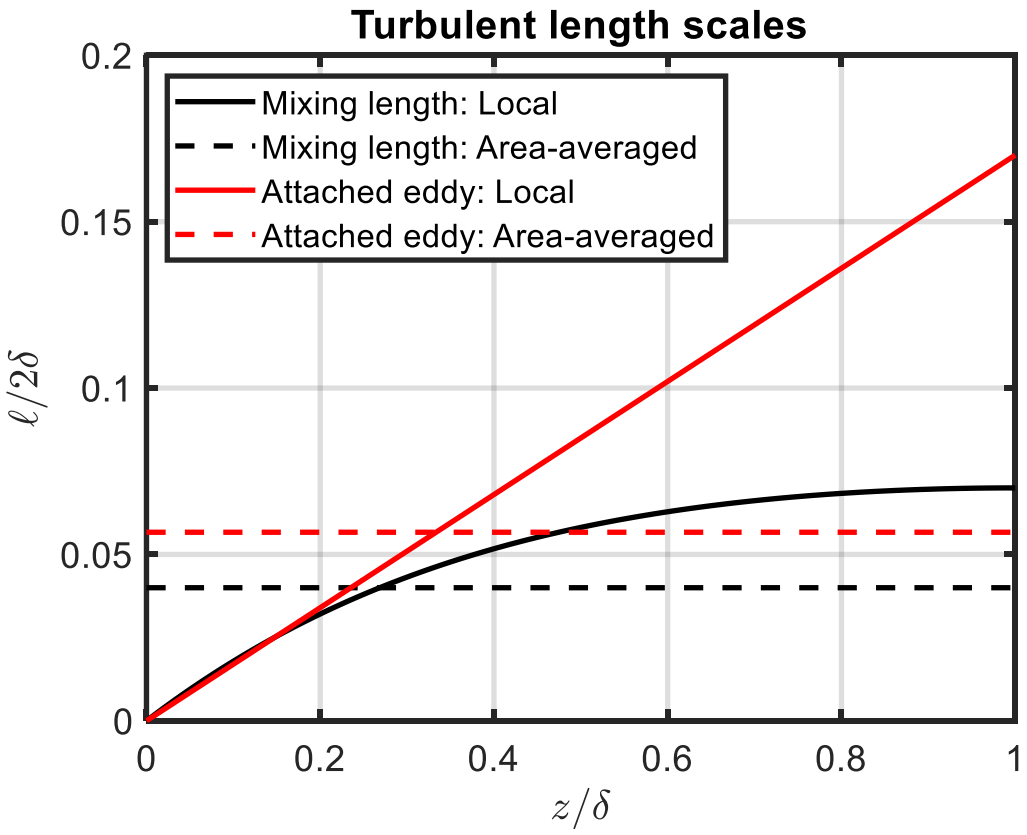
Some protagonists of this philosophy have gone so far as to denounce "the relentless attempts to build into them first-principle content and rational ideas" Spalart (This remark was made in the context of limiting the use of RANS models to the wall-adjacent region in hybrid LES-RANS schemes.),[89] arguing that no RANS model can predict flows with massive separation. Industry at large has preferred simplicity and robustness, even at the expense of physical transparency and comprehensiveness. As a result, CFD software companies have, to a great extent, been responsive to such demands. Some new two-equation and even one-equation eddy-viscosity models have been proposed, tuned for a family of flows particularly suited to certain industrial sectors; indeed, some have become quite popular, despite their containing *ad hoc*, opaque inputs in terms of empirical functions and limiters. One might well conclude that the situation is reminiscent of that of the early days of turbulence modeling a half century ago! Among these, the Spalart-Allmaras (S-A) one-equation model for eddy viscosity (Spalart and Allmaras[101]) and the "shear-stress transport" (SST) k–ω model, Menter[102] are probably the most widely used, at least in the aerospace industry. It seems doubtful that this trend will reverse.

Yet, it *is* recognized that for certain vital applications (whether these be industrial, environmental or medical) *predictive accuracy* still remains of paramount importance. That fact and the increasing geometric complexity of the flow domains to be resolved would seem to favor the employment of second-moment closure. However, even here, the emergence of hybrid LES-RANS approaches (with the RANS region employing an eddy viscosity model) for flows bounded by walls of complex configuration and involving heat and mass transfer offer, for many, the preferred route. So, it may well be the case that differential second-moment closure is destined to remain a relatively minor "niche" market in the firmament of turbulence closure schemes. Nevertheless, there is a reasonable prospect that the better physics embedded in the second-moment approach to closure will, through judicious simplification to algebraic form, still make a significant contribution to enhancing the reliability of CFD for turbulent flows.

Turbulent length scales

- Wall: $z = 0$, center-line (CL): $z = \delta$
- Boundary-layer thickness δ, pipe diameter D:
  - $2\delta = \mathrm{D}$
- Area-averaged (AA):
  - $\langle \cdot \rangle_{AA} = \frac{2}{\delta^2} \int_0^{\delta} [\cdot] \times (\delta - z) dz$

- Mixing-length (ML) [Nikuradse 1933]:
  - $\ell_{ML,CL} = 0.07 \times D$
  - $\ell_{ML,AA} = 0.04 \times D$[Greenshields 2021]
- Attached-eddy (AE) [Townsend 1976] with global (g) von Kármán constant $\kappa_g = 0.34$ [Basse 2021a]:
  - $\ell_{AE,CL} = \frac{\kappa_g}{2} \times D = 0.17 \times D$
  - $\ell_{AE,AA} = \frac{\kappa_g}{6} \times D = 0.06 \times D$

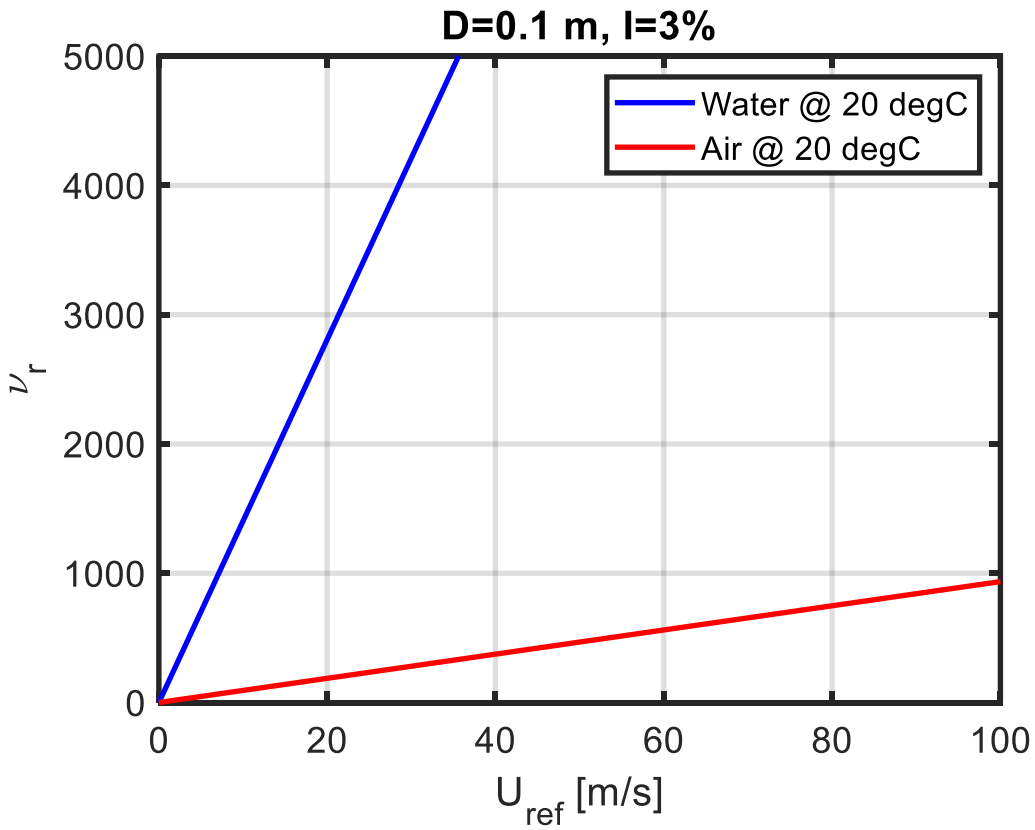


Inlet BCs for $k - \varepsilon$

- k is turbulent kinetic energy (TKE) of velocity fluctuations per unit mass:
  - $k = \frac{3}{2}\overline{u^2} = \frac{3}{2}\left(IU_{ref}\right)^2$
    - Assumes isotropic turbulence
    - $\sqrt{\overline{u^2}}$ is the time-averaged fluctuating velocity
    - U_{ref} is a mean reference velocity
- I is turbulence intensity (TI):
  - $I^2 = \frac{\overline{u^2}}{U_{ref}^2} = \frac{2k}{3U_{ref}^2}$
- ε is rate of dissipation of TKE per unit mass:
  - $\varepsilon = C_\mu^{3/4}\frac{k^{3/2}}{\ell_{ML}}$,
  - $C_\mu = 0.09$ [BFA 1967]
- ν_t is the kinematic turbulent viscosity:
  - $\nu_t = C_\mu \frac{k^2}{\varepsilon} = C_\mu^{1/4}\sqrt{3/2}\boldsymbol{\ell_{ML} I U_{ref}} = 0.67 \times \boldsymbol{\ell_{ML} I U_{ref}}$

Turbulent viscosity ratio

- Total viscosity $\nu_{tot} = \nu_{kin} + \nu_t$
  - ν_{kin} is the kinematic molecular viscosity
- Turbulent viscosity ratio $\nu_r = \dfrac{\nu_t}{\nu_{kin}}$
- Examples:
  - $\nu_t = 0.67 \times \ell_{ML} I U_{ref}$
    $= 0.67 \times (0.07 \times 0.1m) \times (0.03) \times U_{ref}$
    $= 1.4 \times 10^{-4} m \times U_{ref}$
    $\nu_{kin,Water@20degC} = 1.0035 \times 10^{-6} m^2/s$
    $\nu_{kin,Air@20degC} = 15.06 \times 10^{-6} m^2/s$



Motivation

-----Original Message-----
From: CFD Online [mailto:webmaster@cfd-online.com]
Sent: 28. marts 2014 12:08
To: Basse, Nils Taangefjord
Subject: Re: CFD Online Feedback: Original source of equation

I think that this comes from the Fluent and ESI manuals. I am not aware of the original scientific paper. If you find it please let me know.

Yours,

Pete

'On 2014-03-27 13:35, Nils Basse wrote:

> Can you provide the original refernce for the equation relating turbulence intensity and Reynolds number here:
>
> http://www.cfd-online.com/Wiki/Turbulence_intensity

- Started at Siemens:
  - Flowmeter calibration:
    - Scaling of fluctuations with mean flow speed
    - Goal: Provide measurement-based estimates

- CFD Online:
  - ANSYS Fluent scaling (without source→until recently):
    - $I_{CL,[ANSYS]} = \frac{\sqrt{\overline{u_{CL}^2}}}{U_m} = 0.16 \times Re_D^{-1/8}$
      - Bulk Reynolds number $Re_D = \frac{D \times U_m}{\nu_{kin}}, U_m = \langle U \rangle_{AA}$
  - https://www.cfd-online.com/Wiki/Turbulence_intensity

ANSYS Fluent User's Guide

- [ANSYS 2021a]
- [ANSYS 2021b]
- [ANSYS 2022]

For internal flows, the turbulence intensity at the inlets is totally dependent on the upstream history of the flow. If the flow upstream is under-developed and undisturbed, you can use a low turbulence intensity. If the flow is fully developed, the turbulence intensity may be as high as a few percent. The turbulence intensity at the core of a fully-developed duct flow can be estimated from the following formula derived from an empirical correlation for pipe flows:

$$I \equiv \frac{u'}{u_{avg}} = 0.16\left(Re_{D_H}\right)^{-1/8} \tag{7.71}$$

For internal flows, the turbulence intensity at the inlets is totally dependent on the upstream history of the flow. If the flow upstream is under-developed and undisturbed, you can use a low turbulence intensity. If the flow is fully developed, the turbulence intensity may be as high as a few percent. The turbulence intensity at the core of a fully-developed duct flow can be estimated from the following formula derived from an empirical correlation for pipe flows, [165] (p. 4942):

$$I \equiv \frac{u'}{u_{avg}} = 0.16\left(Re_{D_H}\right)^{-1/8} \tag{7.71}$$

For internal flows, the turbulence intensity at the inlets is totally dependent on the upstream history of the flow. If the flow upstream is under-developed and undisturbed, you can use a low turbulence intensity. If the flow is fully developed, the turbulence intensity may be as high as a few percent. The turbulence intensity at the core of a fully-developed duct flow can be derived from an empirical correlation by Blasius for pipe flows (see Chapter 20 Equations (20-4) and (20-5) in [127] (p. 5098)), using the additional estimation of the RMS velocity fluctuation at the pipe core $u' = 0.8 * u_\tau$:

$$I \equiv \frac{u'}{u_{avg}} = 0.16\left(Re_{D_H}\right)^{-1/8} \tag{7.71}$$

ANSYS derivation

- Friction factor $\lambda = \frac{-(\Delta P/L)D}{\frac{1}{2}\rho U_m^2} = 8 \times \frac{u_\tau^2}{U_m^2}$
  - ΔP is pressure loss, L is pipe length and ρ is density
- Friction velocity $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$, τ_w is wall shear stress
- [Blasius 1913] (smooth pipe): $\lambda = 0.3164 \times Re_D^{-1/4}$
- Assume: $\sqrt{\overline{u_{CL}^2}} = 0.8 \times u_\tau$ [TL 1972]
- $I_{CL,[ANSYS]} = \frac{\sqrt{\overline{u_{CL}^2}}}{U_m} = 0.8 \times \sqrt{\frac{\lambda}{8}} = 0.8 \times \sqrt{\frac{0.3164}{8}} \times Re_D^{-1/8} = 0.16 \times Re_D^{-1/8}$

Why is the turbulence intensity important?

- Pipe flow CFD example:
  - Left as an exercise for the reader ☺
- [Momentum] Isothermal:
  - Increase TI:
    - Increased pressure drop
- [Momentum and energy] Conjugate heat transfer (CHT):
  - Assume heat source in pipe walls
  - Increase TI:
    - Increased pressure drop
    - Increased heat transfer coefficient (HTC)
    - Decreased solid temperature

Research findings

- Modelling based on Princeton Superpipe measurements [HVBS 2013]:
  - https://smits.princeton.edu/superpipe-turbulence-data/
  - Note: Weakness to have used only one dataset

- Siemens: [RB 2016]
- Independent Scientist:
  - [Basse 2017]
  - [Basse 2019]
  - [Basse 2021a]
  - [Basse 2021b]

[RB 2016]

- $I_{CL,[RB\ 2016]} = \frac{\sqrt{\overline{u_{CL}^2}}}{U_{CL}} = 0.0550 \times Re_D^{-0.0407}$
- $I_{AM} = \frac{\langle\sqrt{\overline{u^2}}\rangle_{AM}}{U_m} = 0.227 \times Re_D^{-0.100}$

- Arithmetic mean (AM):
  - $\langle\cdot\rangle_{AM} = \frac{1}{\delta}\int_0^{\delta}[\cdot]dz$

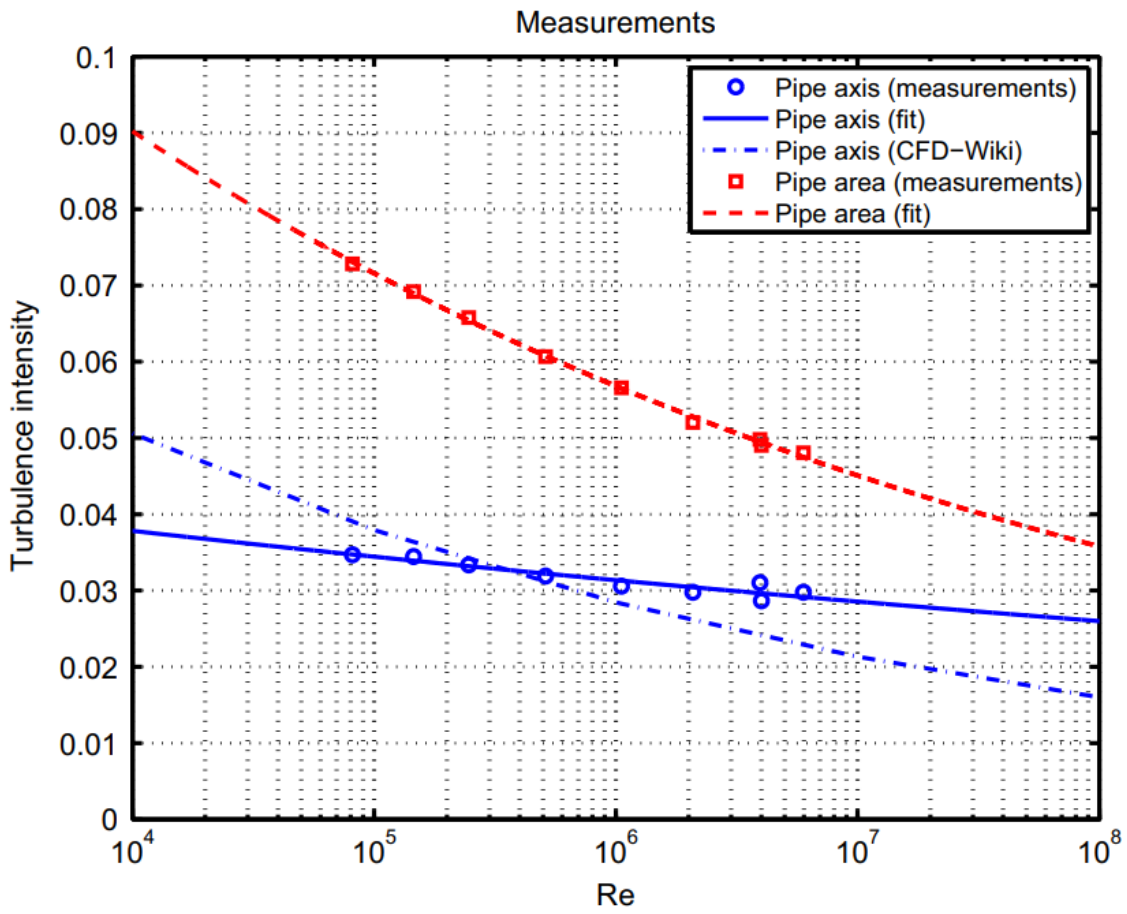


Fig. 22. Turbulence intensity for measured compressible flow.

[Basse 2017]

- $I_{AA} = \frac{\langle\sqrt{\overline{u^2}}\rangle_{AA}}{U_m} = 0.317 \times Re_D^{-0.110}$
- $I \sim \sqrt{\lambda}$ [TL 1972]

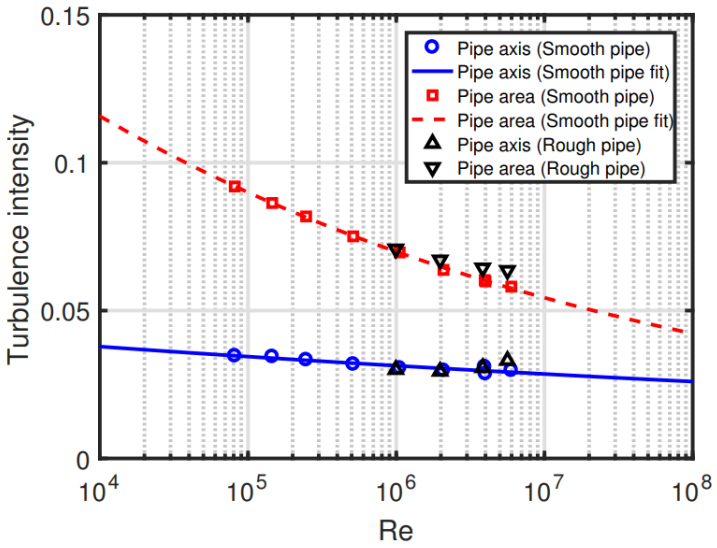


Figure 5. Turbulence intensity for smooth and rough pipe flow.

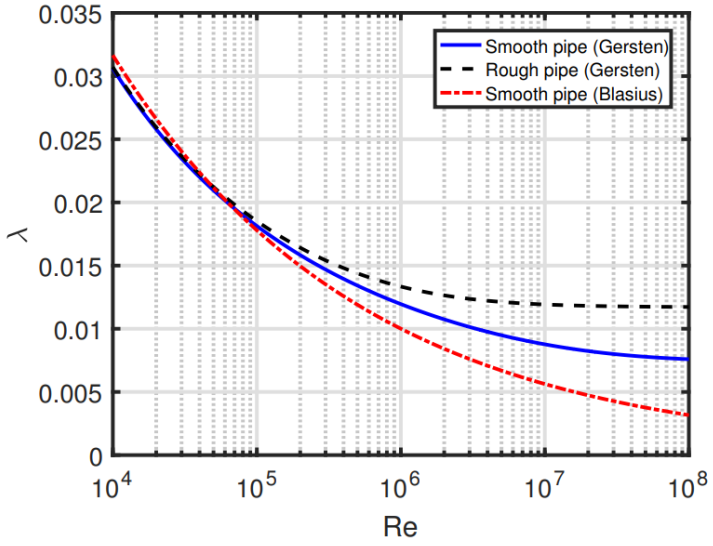


Figure 6. Friction factor.

[Basse 2019]

- Log- and power-law scaling:
  - Both with Re_D and λ as variables
- Example with code:
  - $I_{CL,[Basse\ 2019]} = \frac{\langle\sqrt{\overline{u^2}}\rangle_{AA}}{U_{CL}} = 0.0276 \times \ln(\lambda) + 0.1794$
    - $I_{CL,[Basse\ 2019]} = 0.442 \times \lambda^{0.463}$

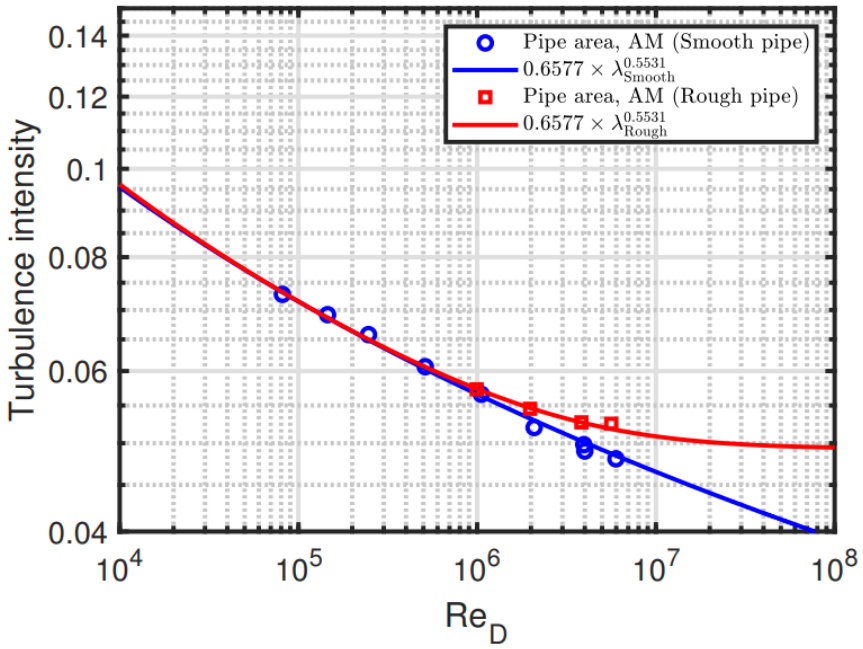


Figure 9. AM definition of TI as a function of Re_D for smooth- and rough-wall pipe flow.

- Useful conversions:
  - $U_{CL} = U_m \times 1.4671 \times Re_D^{-0.0163}$
  - Friction Reynolds number Re_τ:
    - $Re_\tau = \frac{\delta u_\tau}{\nu_{kin}} = \frac{u_\tau}{2U_m} \times Re_D$
    - $Re_\tau = 0.0621 \times Re_D^{0.9148}$

[Basse 2021a]

- Assume global mean velocity log-law:
  - $U_g^+ = \frac{1}{\kappa_g} \times \ln(z^+) + A_g$
    - $U_g^+ = \frac{U_g}{u_\tau}$
    - $z^+ = \frac{z u_\tau}{\nu_{kin}}$
    - Fit parameters κ_g and A_g
- High Reynolds number transition:
  - $Re_\tau \sim 11\,000$
  - $Re_D \sim 5 \times 10^5$
- $\langle I_g^2 \rangle_{AA} = 0.39 \times \lambda$

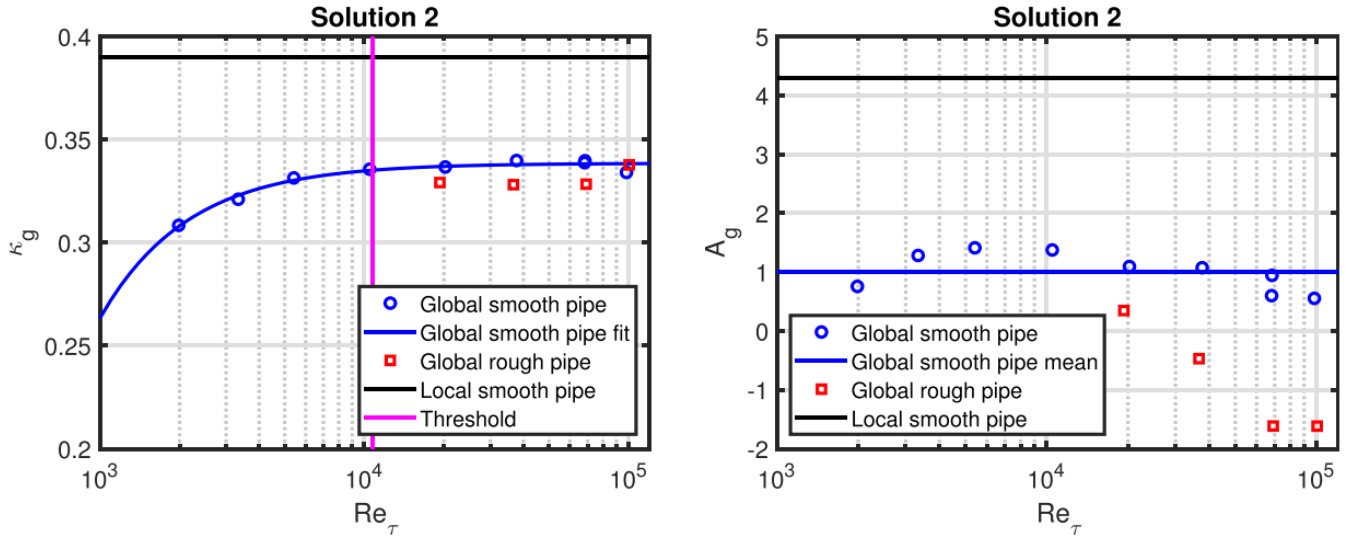


FIG. 10. Left-hand plot: $\kappa_{g,\text{solution 2}}$ vs Re_τ, right-hand plot: $A_{g,\text{solution 2}}$ vs Re_τ. Blue lines are Eqs. (38) and (39), and local parameter values are black lines.

[Basse 2021b]

- Assume global fluctuating velocity log-law:
  - $\overline{u^2}^+ = B_g - A_g \times \ln\left(\frac{z}{\delta}\right) - C_g (z^+)^{-1/2}$
  - $\overline{u^2}^+ = \frac{\overline{u^2}}{u_\tau^2}$
  - Fit parameters B_g, A_g and C_g
- High Reynolds number transition

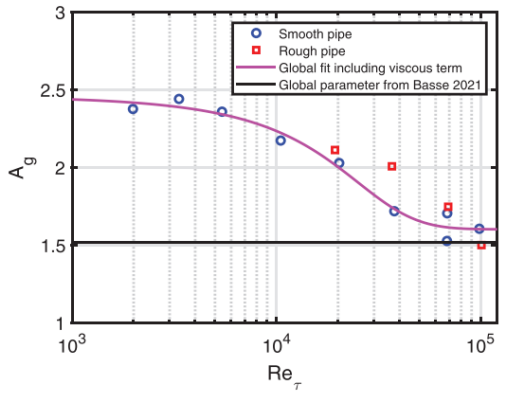


FIG. 6. A_g as a function of Re_τ.

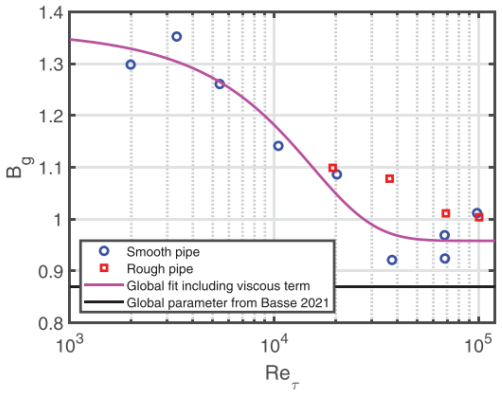


FIG. 7. B_g as a function of Re_τ.

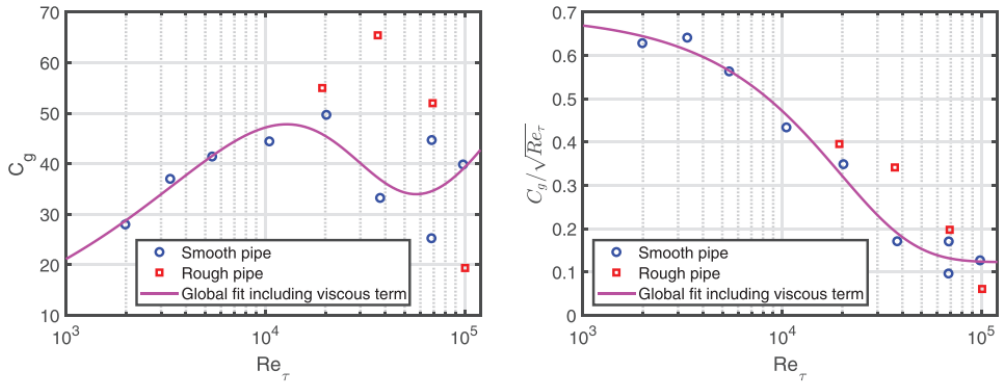
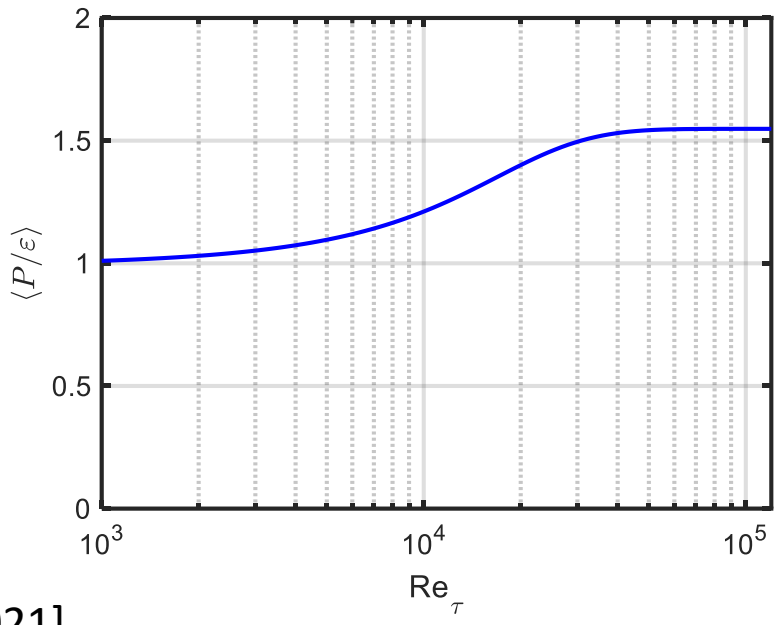


FIG. 8. Left-hand plot: C_g as a function of Re_τ, right-hand plot: $C_g/\sqrt{Re_\tau}$ as a function of Re_τ.

[Basse 2021b]

- Ratio of turbulence production rate P and dissipation rate ε:
  - High Reynolds number transition
  - Possibly related to growth of outer peak



[PRFVO 2021]

Re_τ	T/τ_t	Line style
180.3	204.0	
495.3	87.4	
1136.6	25.9	
1144.2	31.1	NA
1134.6	24.5	NA
1131.0	31.3	NA
1135.7	28.6	NA
1135.7	15.5	NA
1976.0	22.4	
3028.1	16.6	
6019.4	8.32	

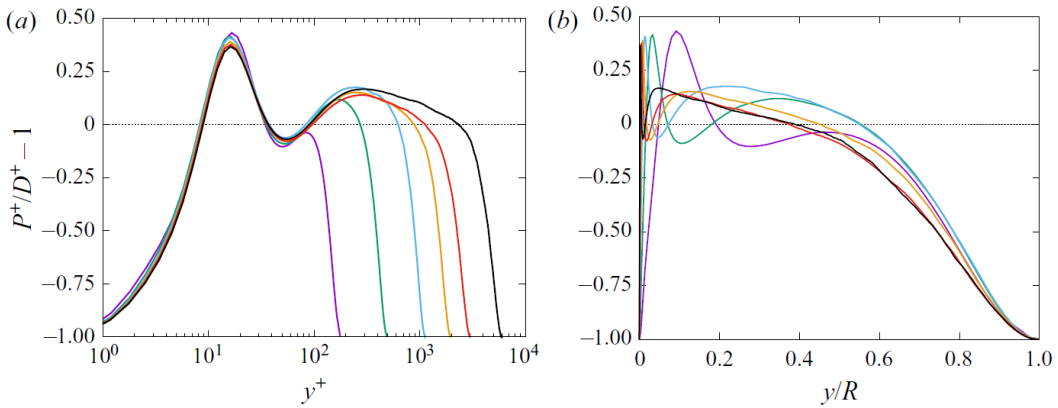
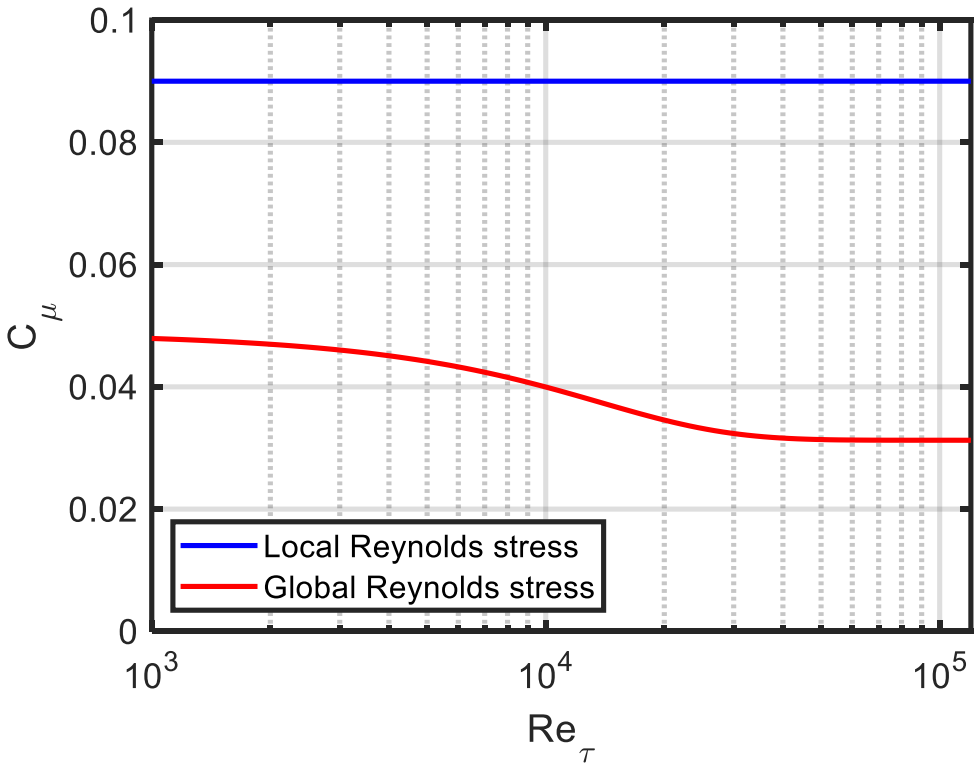


Figure 13. Excess of turbulence kinetic energy production over dissipation as a function of inner-scaled (*a*) and outer-scaled (*b*) wall distance. Lines as in table 1.

[Basse 2022]

- Modelling Reynolds number scaling of C_μ
- Simple turbulent shear flow:
  - $\frac{|\langle uv \rangle|}{k} = \sqrt{C_\mu \frac{P}{\varepsilon}}$
- Local Reynolds stress:
  - $\frac{|\langle uv \rangle|}{k} \approx 0.3$ for $P/\varepsilon \approx 1$
  - $C_\mu = \left(\frac{|\langle uv \rangle|}{k}\right)^2 \frac{\varepsilon}{P} = (0.3)^2 = 0.09$ [BFA 1967]
- Global Reynolds stress:
  - $\frac{|\langle uv \rangle|}{k} \approx \frac{2}{3} \frac{u_\tau^2}{\langle \overline{u^2} \rangle_{AA}} = \frac{2}{3}\left(\frac{1}{1.7277}\right)^2 = 0.22 \simeq \frac{2}{9}$
  - $\langle C_\mu \rangle_{AA} = 0.05 \left\langle \frac{\varepsilon}{P} \right\rangle \simeq \frac{4}{81} \left\langle \frac{\varepsilon}{P} \right\rangle$



Recommendations

- Simple scaling:
  - Power-laws:
    - Smooth pipe
    - Does not capture transition
- More complex scaling:
  - Friction factor:
    - Includes roughness
    - Captures transition

- Use consistent length and velocity scales to calculate ν_t:
  1. Center-line (CL): Lower TI, longer length scale
  2. Area-averaged (AA): Higher TI, shorter length scale

Conclusions and outlook

- Provide measurement-based TI for:
  - General purposes, e.g. calibration
  - BCs for CFD
- Learn new physics along the way:
  - High Reynolds number transition

- Future:
  - TI for other canonical flows: Boundary layers, channels
  - Turbulent length scale and other BCs
  - Overview (anonymous) of BCs used by industry for different applications
    →Close the gap
  - RANS and machine learning (ML)

- Acknowledgment: Professor Alexander Smits and former students

References

- [Roache 1972] P.J.Roache, Computational Fluid Dynamics, Hermosa Publishers, 1972
- [Pope 2000]: S.B.Pope, Turbulent Flows, Cambridge University Press, 2000
- [LS 1974]: B.E.Launder and D.B.Spalding, The Numerical Computation of Turbulent Flows, Computer Methods in Applied Mechanics and Engineering, Vol. 3, pp. 269-289, 1974
- [Runchal 2020] 50 Years of CFD in Engineering Sciences: A Commemorative Volume in Memory of D. Brian Spalding, Springer, 2020
- [HL 2021] K.Hanjalić and B.E.Launder, Reassessment of Modeling Turbulence via Reynolds Averaging: A Review of Second-Moment Transport Strategy, Physics of Fluids Vol. 33, 091302, 2021
- [Nikuradse 1933] J.Nikuradse, Strömungsgesetze in rauhen Rohren, VDI-Verlag, 1933
- [Greenshields 2021] Derivation by C.Greenshields, private communication, 2021
- [Townsend 1976] A.A.Townsend, The Structure of Turbulent Shear Flow, 2nd Ed., Cambridge University Press, 1976

References

- [Basse 2021a] N.T.Basse, Scaling of Global Properties of Fluctuating and Mean Streamwise Velocities in Pipe Flow: Characterization of a High Reynolds Number Transition Region, Physics of Fluids Vol. 33, 065127, 2021
- [BFA 1967] P.Bradshaw et al., Calculation of boundary-layer development using the turbulent energy equation, Journal of Fluid Mechanics Vol. 28, pp. 593-616, 1967
- [ANSYS 2021a] ANSYS Fluent User’s Guide, 2021 R1
- [ANSYS 2021b] ANSYS Fluent User’s Guide, 2021 R2
- [ANSYS 2022] ANSYS Fluent User’s Guide, 2022 R1
- [Blasius 1913] H.Blasius, Das Ähnlichkeitsgesetz bei Reibungsvorgängen in Flüssigkeiten, VDI-Verlag, 1913
- [TL 1972] H.Tennekes and J.L.Lumley, A First Course in Turbulence, MIT Press, 1972
- [HVBS 2013] M.Hultmark et al., Logarithmic scaling of turbulence in smooth- and rough-wall pipe flow, Journal of Fluid Mechanics Vol. 728, pp. 376-395, 2013

References

- [RB 2016] F.Russo and N.T.Basse, Scaling of Turbulence Intensity for Low-Speed Flow in Smooth Pipes, Flow Measurement and Instrumentation Vol. 52, pp. 101-114, 2016
- [Basse 2017] N.T.Basse, Turbulence Intensity and the Friction Factor for Smooth- and Rough-Wall Pipe Flow
- [Basse 2019] N.T.Basse, Turbulence Intensity Scaling: A Fugue, Fluids Vol. 4, 180, 2019
  - Code: https://www.researchgate.net/publication/336374461_Python_code_to_calculate_turbulence_intensity_based_on_Reynolds_number_and_surface_roughness
- [Basse 2021b] N.T.Basse, Scaling of Global Properties of Fluctuating Streamwise Velocities in Pipe Flow: Impact of the Viscous Term, Physics of Fluids Vol. 33, 125109, 2021
- [PRFVO 2021] S.Pirozzoli et al., One-point statistics for turbulent pipe flow up to $Re_\tau \approx 6000$, Journal of Fluid Mechanics Vol. 926, A28, 2021
- [Basse 2022] N.T.Basse, to be submitted for publication, 2022

Useful links

- NASA Turbulence Modeling Resource (aerospace applications, no BC focus):
  - https://turbmodels.larc.nasa.gov/
  - http://www.cfd2030.com/

- NASA Technical Reports Server:
  - [Nikuradse 1933] https://ntrs.nasa.gov/citations/19930093938

- ResearchGate:
  - N.T.Basse
  - K.Hanjalić
  - B.E.Launder

Useful links

- CFD Online:
  - https://www.cfd-online.com/Wiki/Turbulence_length_scale
  - https://www.cfd-online.com/Wiki/Turbulence_intensity

- Markus Uhlmann:
  - http://www-cfd.ifh.uni-karlsruhe.de/uhlmann/VORLESUNG/turbmod/ws08/talk8_ho.pdf
  - http://www-cfd.ifh.uni-karlsruhe.de/uhlmann/VORLESUNG/turbmod/ws08/talk9_ho.pdf
- David D. Apsley:
  - https://personalpages.manchester.ac.uk/staff/david.d.apsley/lectures/comphydr/turbmodel.pdf
- Nils T. Basse:
  - http://www.npb.dk/pub/pub.html