A Study of Multiscale Density Fluctuation Measurements

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Abstract—Intriguing parallels between density fluctuation power versus wavenumber on small (in millimeter) and large (in megaparsec) scales are presented. The comparative study is carried out between fusion plasma measurements and cosmological data. Based on predictions from classical fluid turbulence theory, we argue that our observations are consistent with 2-D turbulence. The similar dependencies of density fluctuations on these disparate scales might indicate that primordial turbulence has been expanded to cosmological proportions.

Index Terms—Cosmology, density fluctuations, fusion plasmas, turbulence, wavenumber spectra.

I. INTRODUCTION

I T IS A VERY human trait to compare new observations to previous experience. Our chance encounter with measurements of the spectral power of density fluctuations on megaparsec scales led us to the conclusion that corresponding millimeter scale measurements in fusion plasmas have surprisingly similar features [1]. We are of the opinion that this correspondence could have a significant impact on current ideas regarding the formation of the universe.

Let us briefly present our reasoning: Fusion plasmas are turbulent, whereas density fluctuations on cosmological scales are not. However, the cosmological fluctuations might be what has been dubbed "fossilized turbulence" [2], [3], i.e., static images of primordial turbulence. This original hot big bang turbulence is in our picture represented by fusion plasma turbulence. So the emerging understanding is as follows: 1) turbulence was generated before the inflationary expansion of the universe, 2) as the universe cooled and expanded, the primordial turbulence fossilized and is visible on cosmological scales today. The theoretical basis of this hypothesis is outlined in [4] and [5].

We show in this paper that both sets of measurements fit the shape expected from 2-D fluid turbulence theory. According to our interpretation, this implies that early turbulence was 2-D.

The fusion plasma measurements presented in this paper are of fluctuations in the electron density. Phase-contrast imaging (PCI) [6] is being used in the Alcator C-Mod tokamak [7] and
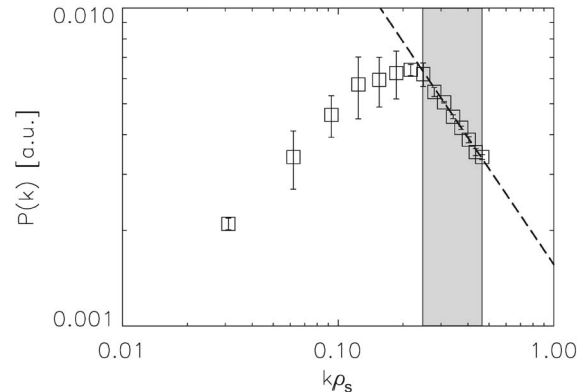
Fig. 1.   Wavenumber spectrum of broadband turbulence in C-Mod. Squares are measured points. The dashed line is a fit to (1); the semitransparent rectangle indicates which points are included to make the fit. The measurements are taken from [13, Fig. 11].

small-angle collective scattering (SACS) [8] was used in the Wendelstein 7-AS (W7-AS) stellarator [9].

We specifically study density fluctuation power P versus wavenumber k (also known as the wavenumber spectrum) in C-Mod and W7-AS. These wavenumber spectra characterize the nonlinear interaction between turbulent modes having different length scales. Our explicit assumption is that turbulence in stellarators and tokamaks is comparable.

The second part of our measurements, a cosmological wavenumber spectrum constructed from a variety of sources, has been published in [10] and was subsequently made available to us [11]. The measurements were used to constrain cosmological variables, e.g., the matter density Ω_{m} and neutrino masses—for further details see [10] and [12].

This paper is organized as follows: In Section II we analyze fusion plasma and cosmological wavenumber spectra. Thereafter we treat the dimensionality of the measurements in Section III. We discuss the hot big bang turbulence theory in Section IV and conclude in Section V.

II. WAVENUMBER SPECTRA

We begin by studying the fusion plasma wavenumber spectrum shown in Fig. 1. The plot shows PCI measurements with a fit to

$$P(k\rho_{\mathrm{s}}) \propto (k\rho_{\mathrm{s}})^{-m} \qquad (1)$$

where ρ_{s} is the ion Larmor radius at the electron temperature and m is a constant. The measurements were made in a low
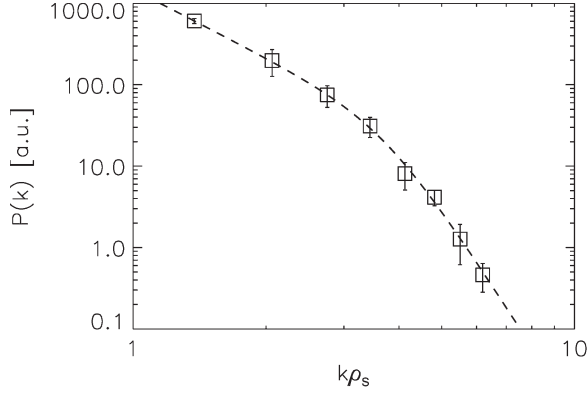
Fig. 2. Wavenumber spectrum of broadband turbulence in W7-AS. Squares are measured points. The dashed line is a fit to (2); all points are included to make the fit. The measurements are taken from [14, Fig. 12].
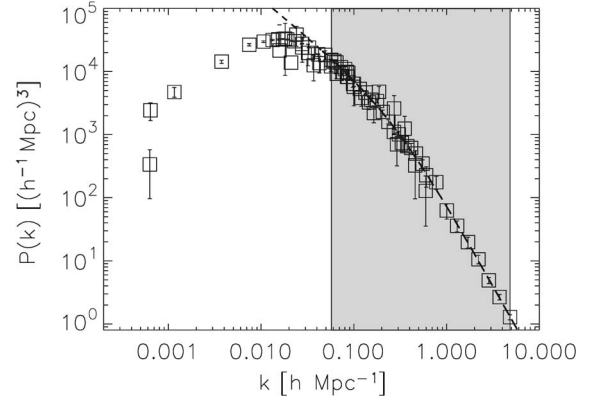

Fig. 3. Wavenumber spectrum of the combined cosmological measurements. Squares are measured points. The dashed line is a fit to (2); the semitransparent rectangle indicates which points are included to make the fit. The measurements are taken from [10, Fig. 38].

confinement mode C-Mod plasma (see [13, Fig. 11]). The wavenumbers measured have been multiplied by ρ_s, which for this case is 0.6 mm. This is the value at 80% of the plasma radius where the electron temperature is 400 eV, the toroidal magnetic field is 6.4 T and the working gas is Deuterium.

Our fit to the indicated PCI data yields $m = 1.0 \pm 0.03$.

All fits shown in this paper have a normalized $\chi^2 \leq 1$, ensuring a satisfactory quality. The error bars are standard deviations and the semitransparent rectangles indicate which points are included to make the fits.

In Fig. 2 we show SACS measurements at somewhat larger wavenumbers compared to the PCI data. Again, the measured wavenumbers have been multiplied by ρ_s, which in this case is 1 mm. This value is also at 80% of the plasma radius where the electron temperature is 300 eV, the toroidal magnetic field is 2.5 T and the working gas is Hydrogen.

The SACS measurements are fitted to

$$P(k\rho_s) \propto \frac{(k\rho_s)^{-p}}{1 + (k\rho_s/(k\rho_s)_0)^q} \qquad (2)$$

where $p = 2.8 \pm 0.6$ and $q = 5.7 \pm 1.3$ are constants. The functional form in (2) is taken from [15]. Basically this equation describes two power-laws, where $P \propto (k\rho_s)^{-p} = (k\rho_s)^{-2.8}$ for medium wavenumbers and $P \propto (k\rho_s)^{-p-q} = (k\rho_s)^{-8.5}$ for large wavenumbers. The transitional $(k\rho_s)_0$ is in our case 3.7. The W7-AS data have been taken from [14, Fig. 12].

It is at this point relevant to note that the medium wavenumber fusion plasma exponent is not always three (or 2.8); it typically varies between three and four depending on specific plasma conditions [16]–[18]. Presumably this is due to different instabilities driving turbulence for varying operating conditions, leading to forcing centered at changing scales.

The cosmological wavenumber spectrum is shown in Fig. 3. The measurements are fitted to (2), but using k instead of $k\rho_s$; in this case, $p = 1.2 \pm 0.1$ and $q = 1.4 \pm 0.05$ are constants. Here, $P \propto k^{-p} = k^{-1.2}$ for small wavenumbers and $P \propto k^{-p-q} = k^{-2.6}$ for medium wavenumbers. The transitional wavenumber k_0 is 0.3 h·Mpc^{-1}. Here, $h = H_0/(100 \text{ km/s/Mpc}) \simeq 0.7$, where H_0 is the Hubble parameter observed today.

III. DIMENSIONALITY OF THE MEASURED FLUCTUATIONS

We begin Section III by summarizing our findings on the dependencies of power on wavenumber in Section II

Small wavenumbers :

$$P(k) \propto k^{-1.0}(\text{fusion}) \text{ or } P(k) \propto k^{-1.2}(\text{cosmology}).$$

Medium wavenumbers;

$$P(k) \propto k^{-2.8}(\text{fusion}) \text{ or } P(k) \propto k^{-2.6}(\text{cosmology}).$$

Large wavenumbers;

$$P(k) \propto k^{-8.5}(\text{fusion}).$$

$$(3)$$

Our measured density fluctuation power is equivalent to the d-dimensional energy spectrum $F_d(k)$ [19]–[21]

$$P(k) = F_d(k) = \frac{E(k)}{A_d}$$

$$A_1 = 2 \qquad A_2 = 2\pi k \qquad A_3 = 4\pi k^2 \qquad (4)$$

where A_d is the surface area of a sphere having radius k and dimension d.

We can convert our results in (3) either under the 2-D turbulence assumption

Small wavenumbers :

$$E(k) \propto k^{0.0}(\text{fusion}) \text{ or } E(k) \propto k^{-0.2}(\text{cosmology}).$$

Medium wavenumbers :

$$E(k) \propto k^{-1.8}(\text{fusion}) \text{ or } E(k) \propto k^{-1.6}(\text{cosmology}).$$

Large wavenumbers :

$$E(k) \propto k^{-7.5}(\text{fusion})$$

$$(5)$$

or under the 3-D turbulence assumption

$$\text{Small wavenumbers}:$$

$$E(k) \propto k^{1.0}(\text{fusion}) \text{ or } E(k) \propto k^{0.8}(\text{cosmology}).$$

$$\text{Medium wavenumbers}:$$

$$E(k) \propto k^{-0.8}(\text{fusion}) \text{ or } E(k) \propto k^{-0.6}(\text{cosmology}).$$

$$\text{Large wavenumbers}:$$

$$E(k) \propto k^{-6.5}(\text{fusion}).$$

$$(6)$$

The established picture of 2-D fluid turbulence is: 1) turbulence is forced on an intermediate scale $k_{\text{f(2D)}}$, 2) energy is transferred to larger scales by the inverse energy cascade, $E(k) \propto k^{-5/3}$ [22], enstrophy is transferred to smaller scales by the forward enstrophy cascade, $E(k) \propto k^{-3}$ [23], and 3) enstrophy is dissipated at the smallest scales [24].

For 3-D turbulence the following process occurs: 1) turbulence is forced on a large scale $k_{\text{f(3D)}}$, 2) energy is transferred to smaller scales by the forward energy cascade, $E(k) \propto k^{-5/3}$, and 3) energy is dissipated at the smallest scales.

It is interesting to note that in [25], two dependencies of the 3-D energy spectrum on wavenumber in the dissipation range are considered: One is an exponential falloff, while the other claims that $E(k) \propto k^{-7}$ and was proposed by W. Heisenberg. This power-law is quite close to the one we found for fusion plasmas at large wavenumbers.

To determine whether 2-D or 3-D turbulence is observed, we consider the power-laws for medium wavenumbers: The exponents should roughly be in the range $[-3, -5/3]$ for 2-D turbulence and about $-5/3$ for 3-D turbulence. Equations (5) and (6) indicate that the observed 2-D slopes are close to the expected power-laws and that the 3-D slopes are too shallow. We note that the 2-D slopes are closer to the value for the inverse energy cascade than for the forward enstrophy cascade. The reason for this is not understood.

Turbulence in fusion plasmas is approximately 2-D, since transport along magnetic field lines is nearly instantaneous. For this reason, fluctuations are measured parallel to the major radius of the machine, i.e., perpendicular to the confining magnetic field.

The reason we chose to analyze fusion plasma data was simply a matter of having available measurements and expertise in that field. Any turbulent 2-D plasma should display similar characteristics.

IV. Hot Big Bang Turbulence

In [1] we suggested that the observed plasma turbulence might originate during an early phase in the formation of the universe. Recent theoretical work on the role of hot big bang turbulence in the primordial universe [5] lends support to this assumption:

In this theory, turbulence observed today was created before cosmological inflation by inertial-vortex forces leading to an inverse big-bang turbulence cascade with a -5/3 power-law exponent. Turbulence in the plasma epoch has low Reynold's numbers $\sim 10^2$ according to this picture and the preceding quark-gluon plasma has a large gluon viscosity that acts to damp the big bang turbulence. The claim is that hot big bang temperature turbulence is fossilized before the universe cools to the Grand Unified Theory strong force freeze-out temperature 10^{28} K. Later, during nucleosynthesis, fossil temperature turbulence was converted to fossil turbulence patterns in, e.g., density turbulence [4].

Analysis of power spectra of cosmic microwave background radiation temperature anisotropies shows that they most likely have a turbulent origin, supporting the idea of turbulence generated in the big bang or the plasma epoch [26].

V. Conclusion

The fact that density fluctuations on small (fusion plasma) and large (cosmological) scales can be described by similar functional dependencies, approximately consistent with 2-D fluid turbulence, might indicate that (plasma) turbulence at early times has been fossilized and expanded to cosmological proportions.

Our conjecture concerning the primordial turbulence reflected in our wavenumber spectra can be described as follows: Forcing occurs at an intermediate scale $k_{\text{f(2D)}}$. The inverse energy cascade leads to spectral condensation at large scales and the forward enstrophy cascade leads to enstrophy transfer toward smaller scales. At very small scales, enstrophy is dissipated.

Our interpretation of the measured wavenumber spectra is consistent with the theoretical framework on hot big bang turbulence presented in [5] and references therein.

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