Friction-based scaling of streamwise turbulence intensity in zero-pressure-gradient and pipe flows

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Abstract

We explore the analogy between asymptotic scaling of two canonical wall-bounded turbulent flows, i.e. zero-pressure-gradient and pipe flows; we find that these flows can be characterised using similar scaling laws which relate streamwise turbulence intensity and friction.

1. Introduction

A recent paper [1] on zero-pressure-gradient (ZPG) flow has introduced an asymptotic (high Reynolds number) scaling law:

$$\tilde{U_\tau} \sim \frac{1}{\sqrt{\tilde{\delta}}}, \tag{1}$$

where

$$\tilde{U_\tau} = \frac{U_\tau \nu}{M} \sim \frac{\nu}{U_\tau \delta} = \frac{1}{Re_\tau} \tag{2}$$

is named the "dimensionless drag" and

$$\tilde{\delta} = \frac{\delta M}{\nu^2} \sim \frac{\delta^2 U_\tau^2}{\nu^2} = Re_\tau^2 \tag{3}$$

scales as the friction Reynolds number (Re_τ) squared. Note that we use $\sim$ to mean "scales as". Here, U_τ is the friction velocity, $M = \int_0^\delta U^2 \mathrm{d}z$ is the

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kinematic momentum rate through the boundary layer, ν is the kinematic viscosity, δ is the boundary layer thickness, z is the distance from the wall and U is the mean velocity in the streamwise direction. Note the asymptotic scaling $M \sim U_\tau^2 \delta$ has been proposed in [1] and applied in Equations (2) and (3).

In this paper we will show that the product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ - when a correction term is included in Equation (1) - scales as the turbulence intensity (TI) I [2, 3, 4]. As a consequence, the squared product $(\tilde{U}_\tau^2 \tilde{\delta})$ scales as the friction factor λ. We note that drag was addressed in [1]; the TI was not discussed.

Our study employs global averaging; similarities between local streamwise TI for the same canonical flows have been treated in e.g. [5, 6].

The paper is organized as follows: In Section 2, we briefly review results from asymptotic scaling of TI in pipe flows; these findings are related to ZPG flows in Section 3 and we conclude in Section 4.

2. Asymptotic pipe flow scaling of the streamwise turbulence intensity

The material in this section is a summary of research contained in [2, 3, 4]. The local (streamwise) TI is defined as:

$$I_{\text{local}}(r) = \frac{U_{\text{RMS}}(r)}{U(r)}, \tag{4}$$

where r is the radius ($r = 0$ is the pipe axis and $r = R$ is the pipe wall) and $U_{\text{RMS}}(r)$ is the local root-mean-square (RMS) of the turbulent streamwise velocity fluctuations. The local TI can then be used to define a global TI:

$$I_{\text{global}} = \langle I_{\text{local}}(r) \rangle, \tag{5}$$

where $\langle \cdot \rangle$ indicates radial averaging; see [4], where several definitions of radial averaging have been documented. In the remainder of this paper, we treat the global TI; for simplicity of notation, we drop the subscript "global" and refer to I instead of I_{global}.

For pipe flow, the streamwise turbulence intensity I_{pipe} scales roughly with the ratio of the friction and mean velocities [3, 4]:

$$I_{\text{pipe}} \sim \frac{U_\tau}{U_m} = 2 \times \frac{Re_\tau}{Re_D}, \tag{6}$$

where U_m is the mean velocity and $Re_D = DU_m/\nu$ is the bulk Reynolds number based on the pipe diameter D. For pipe flow, $Re_\tau = RU_\tau/\nu$. The pipe friction factor λ_{pipe} scales with the square of this ratio:

$$\lambda_{\text{pipe}} = 8 \times \frac{U_\tau^2}{U_m^2} = 32 \times \frac{Re_\tau^2}{Re_D^2} \tag{7}$$

As a consequence, the streamwise turbulence intensity scales with the square root of the friction factor:

$$I_{\text{pipe}} \sim \sqrt{\lambda_{\text{pipe}}} \tag{8}$$

An example of the scaling using Princeton Superpipe measurements [7, 8] is Equation (23) in [4]:

$$I_{\text{pipe area, AM}} = 0.6577 \times \lambda_{\text{pipe}}^{0.5531}, \tag{9}$$

where AM is an abbreviation for the "arithmetic mean" radial averaging.

3. Equivalence between zero-pressure-gradient and pipe flows

In [9], we have used the "log law" for the streamwise mean velocity [6] to derive a correction term $\sqrt{f(Re_\tau)}$ for the asymptotic scaling of drag presented in Equation (1):

$$\tilde{U}_\tau \times \sqrt{f(Re_\tau)} \sim \frac{1}{\sqrt{\tilde{\delta}}}, \tag{10}$$

where

$$\sqrt{f(Re_\tau)} = \sqrt{\frac{2}{\kappa^2} - \frac{2A}{\kappa} + A^2 + \log(Re_\tau)\left(\frac{2A}{\kappa} - \frac{2}{\kappa^2}\right) + \log(Re_\tau)^2/\kappa^2} \tag{11}$$

Here, $\kappa = 0.39$ (von Kármán constant) and $A = 4.3$ are constants provided in [6] for fits to smooth wall measurements. The ZPG measurements are fitted to:

$$\sqrt{f(Re_\tau)} = C \times \tilde{\delta}^D, \tag{12}$$

and the pipe measurements are fitted to:

$$\sqrt{f(Re_\tau)} = C \times \lambda_{\text{pipe}}^D, \tag{13}$$

where C and D are fit parameters, see Table 1 and Figure 1.

Table 1: Fit parameters and coefficient of determination (R^2) for Equations (12) and (13).

Power-law constants	C	D	R^2
Equation (12)	7.50	0.049	0.995
Equation (13)	3.08	-0.49	0.998

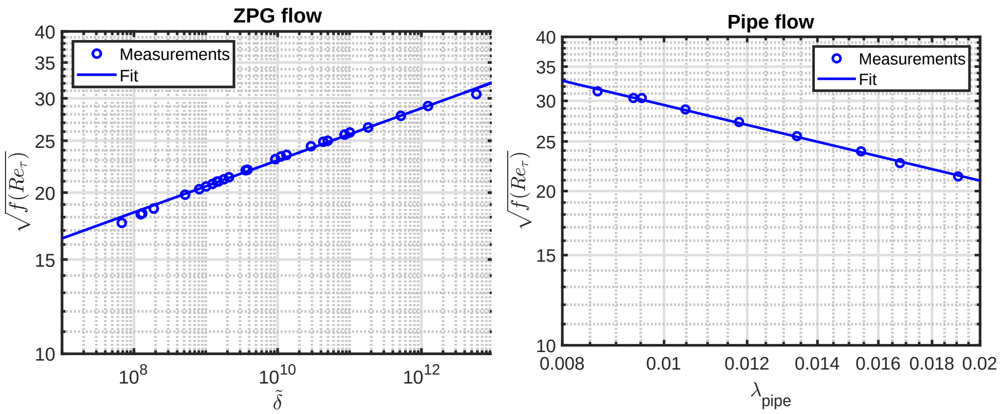

Figure 1: Left-hand plot: Correction term for ZPG flow as a function of $\tilde{\delta}$, right-hand plot: Correction term for pipe flow as a function of λ_{pipe}.

For ZPG flow, we combine Equations (10) and (12):

$$\sqrt{f(Re_\tau)} \sim \frac{1}{\tilde{U}_\tau \sqrt{\tilde{\delta}}} \sim \tilde{\delta}^{0.049} \tag{14}$$

For pipe flow, we combine Equations (8) and (13):

$$\sqrt{f(Re_\tau)} \sim \lambda_{\text{pipe}}^{-0.49} \sim \frac{1}{I_{\text{pipe}}} \tag{15}$$

Since the correction terms in Equations (14) and (15) are equal (the log law applies to both ZPG and pipe flow), we arrive at:

$$I_{\text{pipe}} \sim \tilde{U}_\tau \sqrt{\tilde{\delta}} \tag{16}$$

and:

$$\lambda_{\text{pipe}} \sim \tilde{\delta}^{-0.1} \tag{17}$$

Alternatively, we can combine Equations (8) and (16) to write:

$$\lambda_{\text{pipe}} \sim I_{\text{pipe}}^2 \sim \tilde{U}_\tau^2 \tilde{\delta} \tag{18}$$

By comparing Equations (17) and (18) we see that:

$$\tilde{U}_\tau \sim \tilde{\delta}^{-0.55}, \tag{19}$$

which is consistent with the "discrete" model, see Equations (7) and (8) in [1]. Finally, by using Equations (3) and (19), we can express the product $\tilde{U}_\tau \sqrt{\tilde{\delta}}$ as a function of Re_τ:

$$\tilde{U}_\tau \sqrt{\tilde{\delta}} \sim \tilde{\delta}^{-0.05} \sim Re_\tau^{-0.1}, \tag{20}$$

which is scaling behaviour similar to what has been observed in pipe flow [2, 3, 4]. The exact fit to Equation (20) yields $\tilde{U}_\tau \sqrt{\tilde{\delta}} = 0.10 \times Re_\tau^{-0.11}$ with $R^2 = 0.964$, see Figure 2.

Based on the findings in this paper we summarise the following TI analogies for ZPG and pipe flows:

$$I_{\text{pipe}} \sim \frac{1}{\sqrt{f(Re_\tau)}} \sim \tilde{U}_\tau \sqrt{\tilde{\delta}} \sim I_{\text{ZPG}} \tag{21}$$

Corresponding friction factor analogies can be found by taking the square of Equation (21):

$$\lambda_{\text{pipe}} \sim \frac{1}{f(Re_\tau)} \sim \tilde{U}_\tau^2 \tilde{\delta} \sim \lambda_{\text{ZPG}} \tag{22}$$

4. Conclusions

We have explored the correspondence between zero-pressure-gradient (ZPG) and pipe flows for asymptotic scaling of streamwise turbulence intensity with friction. It is demonstrated that similar scalings are valid for both types of

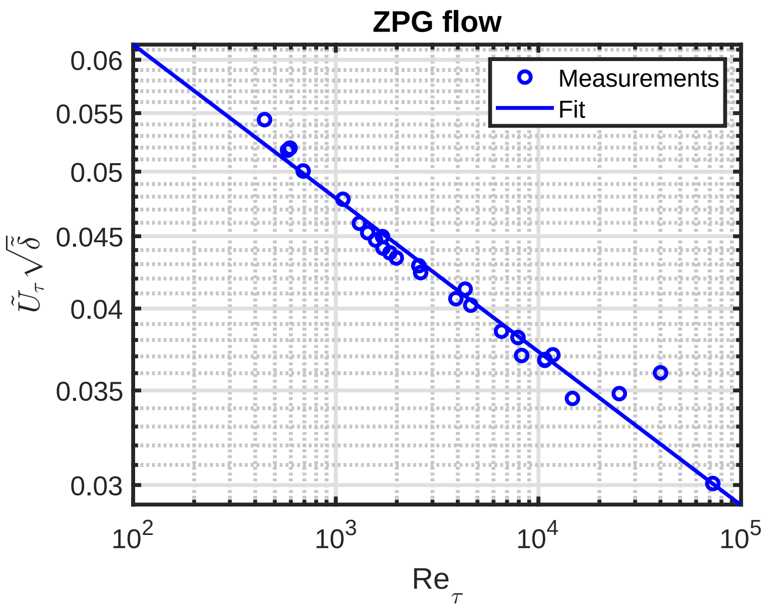

Figure 2: The product $\tilde{U}_\tau\sqrt{\tilde{\delta}}$ as a function of Re_τ.

flows; the product $\tilde{U}_\tau\sqrt{\tilde{\delta}}$ for ZPG flow is equivalent to the streamwise turbulence intensity for pipe flow I_{pipe}. The scaling of turbulence intensity with Reynolds number in ZPG flow closely matches the corresponding pipe flow scaling. In addition, we have shown that the turbulence intensity is inversely proportional to the correction term $\sqrt{f(Re_\tau)}$.

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Data availability statement. Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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