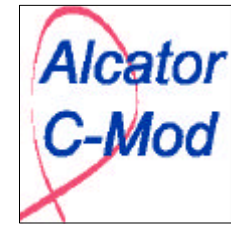


A study of multiscale density fluctuations



N.P. Basse¹
Plasma Science and Fusion Center
Massachusetts Institute of Technology
Cambridge, MA 02139
USA

¹Present address:

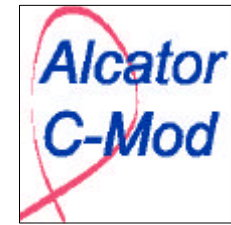
ABB Switzerland Ltd.
Corporate Research
Baden-Dättwil, CH-5405
Switzerland

33rd IEEE International Conference on Plasma
Science, Traverse City, Michigan, USA (2006)

Work supported by US DoE Office of Fusion Energy Sciences



Introduction

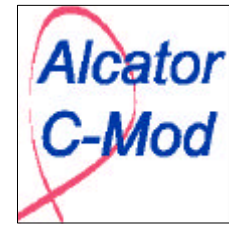


- Intriguing parallels between density fluctuation power versus wavenumber on small (mm) and large (Mpc) scales are presented. The comparative study is carried out between fusion plasma measurements and cosmological data^{1,2}.
- Our observations are consistent with classical 2D fluid turbulence.
- The similar dependencies of density fluctuations on these disparate scales might indicate that primordial turbulence has been expanded to cosmological proportions.

¹N.P. Basse, “Density fluctuations on mm and Mpc scales”, Phys. Lett. A 340 (2005) 456-460.

²N.P. Basse, “A study of multiscale density fluctuation measurements”, submitted (2006), astro-ph/0602452.

Wavenumber spectra



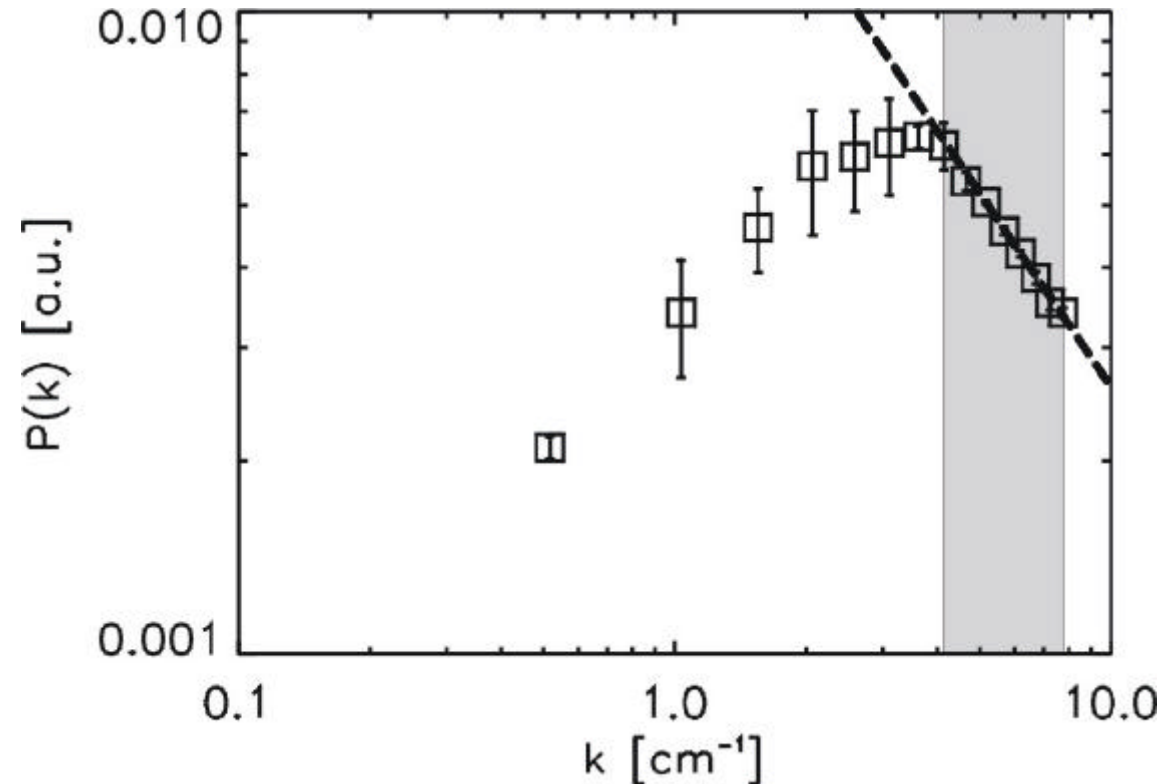
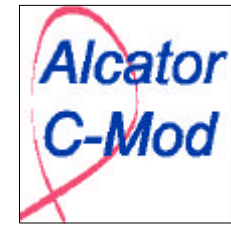
Density fluctuation power P versus wavenumber k (also known as the wavenumber spectrum) is fitted using the following functions:

1. $P(k) \propto k^{-m}$, m is a constant
2. $P(k) \propto (1/k^a) \times e^{-nk}$, a and n are constants
3. $P(k) \propto k^{-p}/[1+(k/k_0)^q]$, p , q and k_0 are constants

All fits shown have $\chi^2 = 1$, ensuring a satisfactory quality.

The error bars are standard deviations and the semi-transparent rectangles indicate which points are included to make the fits.

Fusion: Small wavenumbers

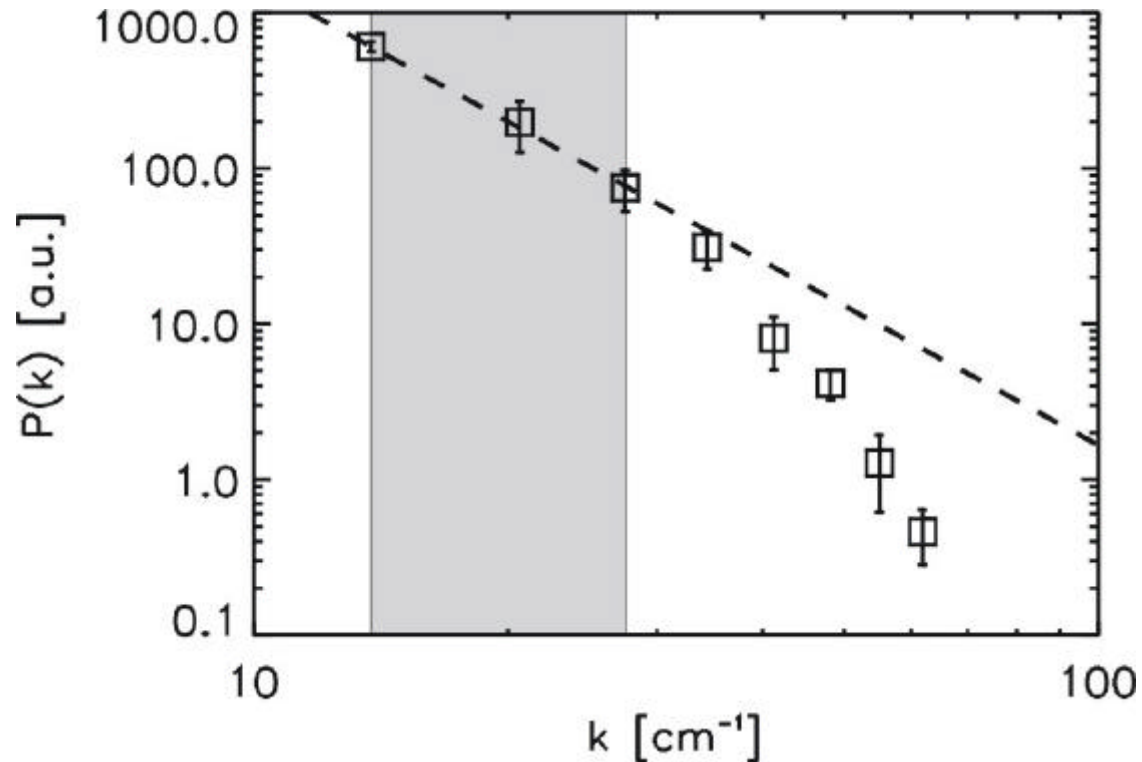
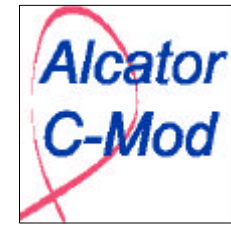


A wavenumber spectrum from the Alcator C-Mod tokamak³ is shown along with a fit to Eq. (1):

$$m = 1.0 \pm 0.03$$

³N.P. Basse et al., Phys. Plasmas 12 (2005) 052512.

Fusion: Medium wavenumbers

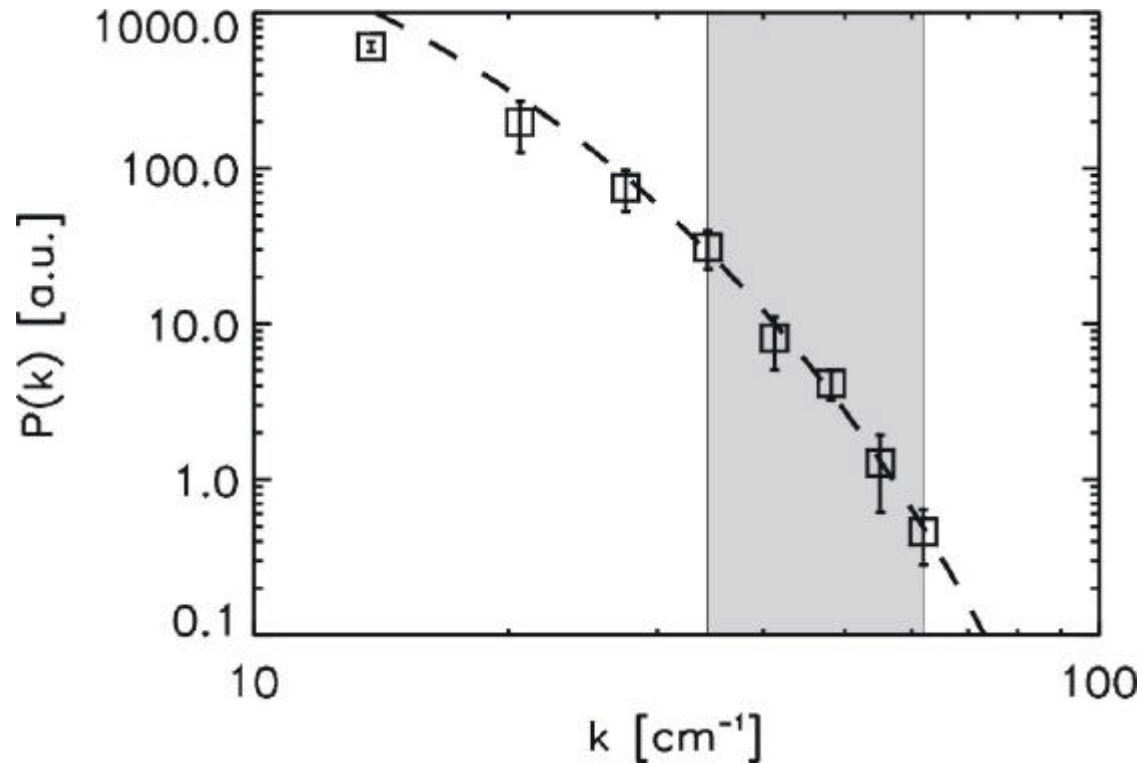
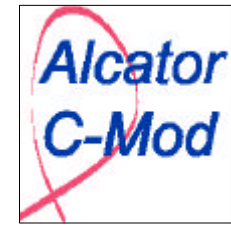


A wavenumber spectrum from the Wendelstein 7-AS stellarator⁴ is shown along with a fit to Eq. (1):

$$m = 2.8 \pm 0.4$$

⁴N.P. Basse et al., Phys. Plasmas 9 (2002) 3035.

Fusion: Large wavenumbers

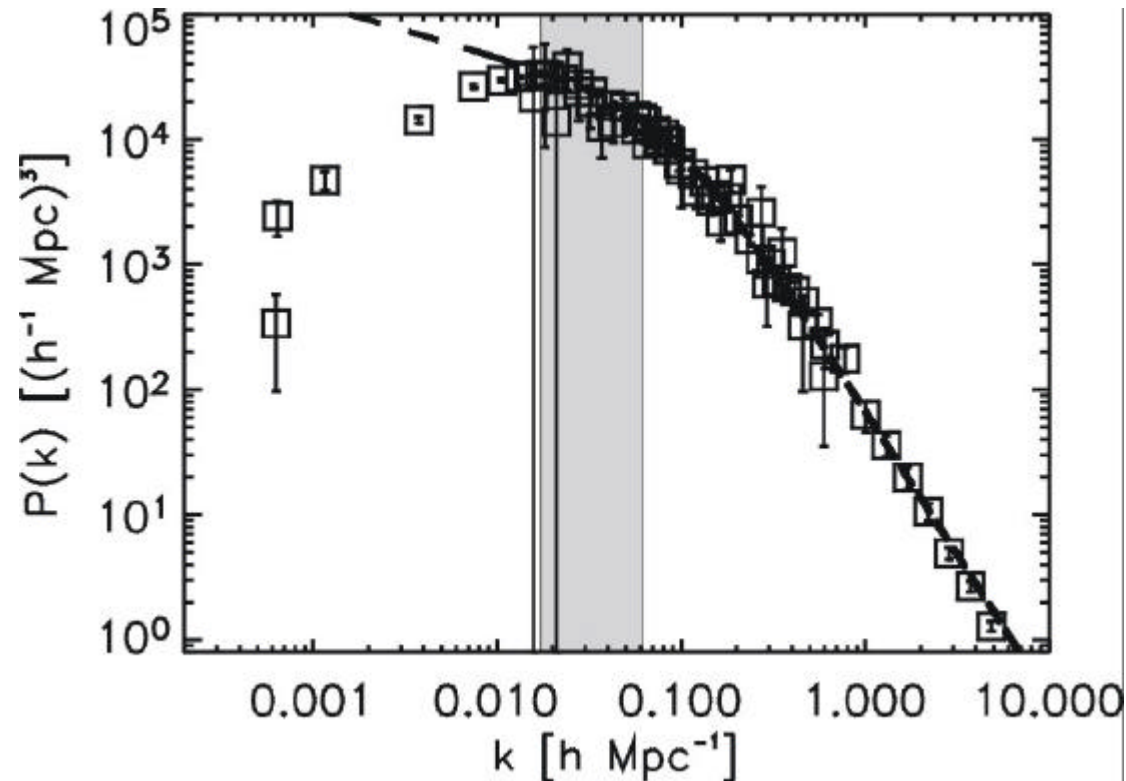
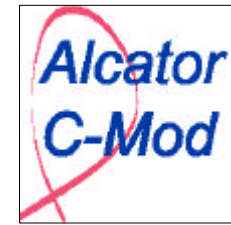


A wavenumber spectrum from the Wendelstein 7-AS stellarator⁴ is shown along with a fit to Eq. (2):

$$a = 1.1 \pm 0.1, n = 0.13 \pm 0.007 \text{ cm}$$

⁴N.P. Basse et al., Phys. Plasmas 9 (2002) 3035.

Cosmology: Small and medium wavenumbers

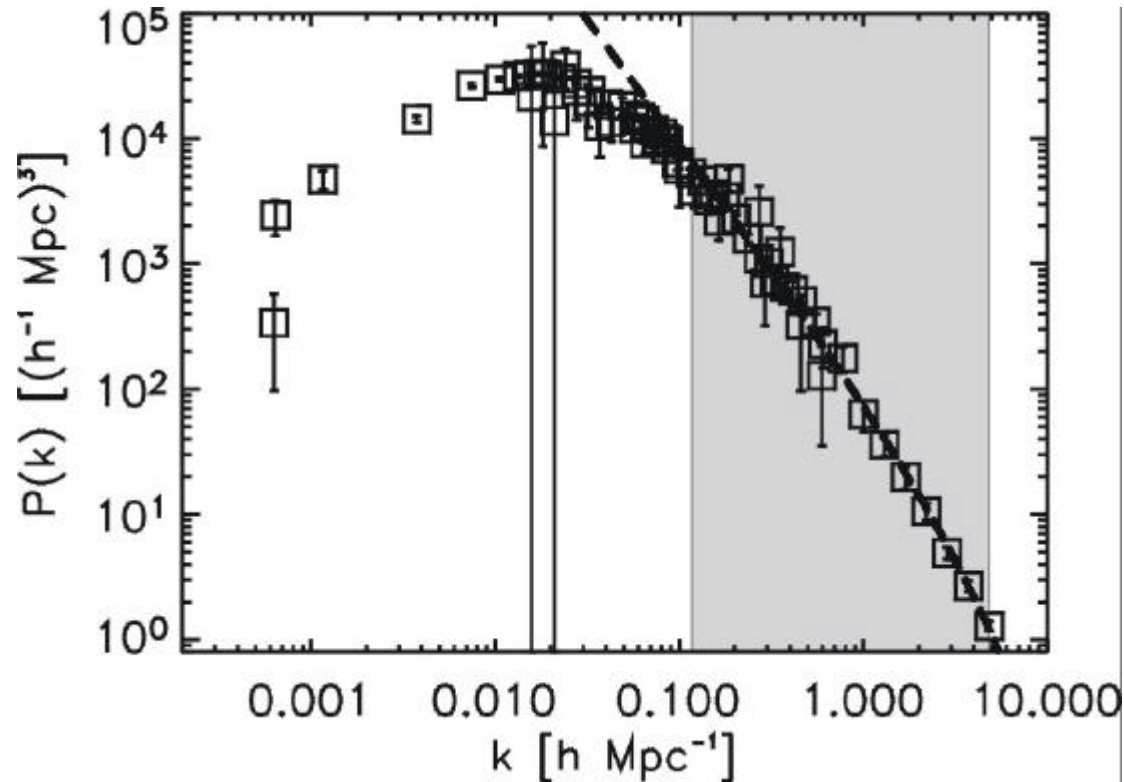
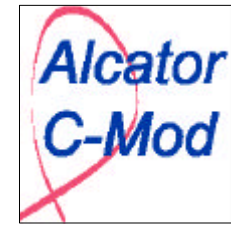


A cosmological wavenumber spectrum from a variety of sources⁵ is shown along with a fit to Eq. (3):

$$p = 0.4 \pm 0.001, q = 1.9 \pm 0.01, k_0 = 0.09 h \text{ Mpc}^{-1}$$

⁵M. Tegmark et al., *Astrophys. J.* 606 (2004) 702.

Cosmology: Large wavenumbers

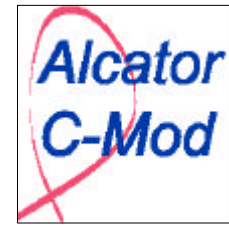


A cosmological wavenumber spectrum from a variety of sources⁵ is shown along with a fit to Eq. (2):

$$n = 0.26 \pm 0.00001 \text{ h}^{-1} \text{ Mpc}, a = 2.0 \pm 0.00001$$

⁵M. Tegmark et al., *Astrophys. J.* 606 (2004) 702.

Summary of findings



Small wavenumbers:

$P(k) \propto k^{-1.0}$ (fusion) or $P(k) \propto k^{-0.4}$ (cosmology)

Medium wavenumbers:

$P(k) \propto k^{-2.8}$ (fusion) or $P(k) \propto k^{-2.3}$ (cosmology)

Large wavenumbers:

$P(k) \propto (1/k^a) \times e^{-nk}$, $a = 1.1$ (fusion) or $a = 2.0$ (cosmology)

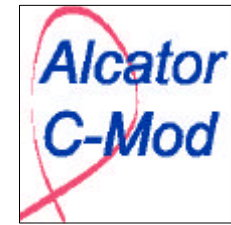
The energy spectrum $E(k)$ is related to $P(k)$ through

$$E(k) = A_d \times P(k),$$

where A_d is the surface area of a sphere having radius k and dimension d .

We assume that the observed fluctuations are 2D ($d=2$), see Ref. [2] for a discussion on the dimensionality of the measurements.

2D turbulence assumption



Small wavenumbers:

$$E(k) \propto k^{0.0} \text{ (fusion) or } E(k) \propto k^{0.6} \text{ (cosmology)}$$

Medium wavenumbers:

$$E(k) \propto k^{-1.8} \text{ (fusion) or } E(k) \propto k^{-1.3} \text{ (cosmology)}$$

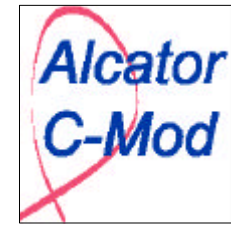
Large wavenumbers:

$$E(k) \propto (1/k^{a-1}) \times e^{-nk}, \quad a = 1.1 \text{ (fusion) or } a = 2.0 \text{ (cosmology)}$$

2D fluid turbulence picture:

- Turbulence is forced on an intermediate scale $k_{f(2D)}$
- Energy is transferred to larger scales by the inverse energy cascade,
 $E(k) \propto k^{-5/3}$ (**medium-small wavenumbers**)
- Energy is transferred to smaller scales by the forward enstrophy cascade,
 $E(k) \propto k^{-3}$ (**medium-large wavenumbers**)
- Energy is dissipated at the smallest scales (**large wavenumbers**)

Hot big bang turbulence



Recent theoretical work on the role of hot big bang turbulence in the primordial universe⁶ supports the conjecture that fusion plasma turbulence represents the physical conditions during either (i) the big bang or (ii) the subsequent plasma epoch.

According to the theory, fluctuations observed today were created before inflation, leading to an inverse big bang turbulence cascade with a $-5/3$ power-law exponent.

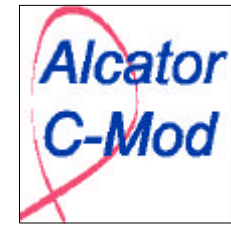
The hot big bang temperature turbulence was fossilized⁷ and converted to fossil turbulence patterns in e.g. density fluctuations.

Using the sketch above, our fusion measurements represent turbulence in the early universe and the cosmological data is equivalent to the fossilized remains of that original turbulence.

⁶C.H. Gibson, Combust. Sci. Tech. 177 (2005) 1049.

⁷G. Gamow, Proc. Natl. Acad. Sci. 40 (1954) 480.

Conclusions



- The fact that density fluctuations on small (fusion plasma) and large (cosmological) scales can be described by wavenumber dependencies consistent with 2D fluid turbulence might indicate that (plasma) turbulence at early times has been fossilized and expanded to cosmological proportions.
- Our conjecture can be described as follows: Forcing occurs at an intermediate scale $k_{f(2D)}$. The inverse energy cascade leads to spectral condensation⁸ at large scales and the forward enstrophy cascade leads to energy transfer towards smaller scales. At very small scales, energy is dissipated.
- The theoretical framework on hot big bang turbulence presented in Ref. [6] is consistent with our interpretation of the measured wavenumber spectra.

⁸M.G. Shats et al., Phys. Rev. E 71 (2005) 046409.

We thank M. Tegmark for providing the cosmological measurements.

- More information: E-mail nils.basse@ch.abb.com, homepage www.npb.dk.